# Physics of Graphene and its Multilayers: From Zero-Mode Anomalies to Band-Gap Opening

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The purpose of this paper is to give a brief review on characteristic features of electronic states in graphene and its multi-layers mainly from a theoretical point of view, together with topics on roles of electron-phonon interactions studied recently.

The electron motion in graphene is governed by Weyl's equation for a neutrino or the Diracequation with vanishing rest mass characterized by a velocity which is about 1/300 of the light velocity. The pseudo-spin wave function exhibits a sign change due to Berry's phase when the wave vector $\boldsymbol{k}$ is rotated around the origin and therefore has a topological singularity at $\boldsymbol{k} = 0$. This singularity is the origin of the peculiar behavior in the transport properties of graphene, such as the minimum conductivity in the absence of a magnetic field, the quantum Hall effect, and the dynamical conductivity [1], as well as the absence of backscattering in metallic carbon nanotubes [2–4]. The neutrino equation is invariant under special time-reversal operation $S$ corresponding to the system with strong spin-orbit interaction. Because $S$ is not the real time-reversal symmetry, it can be destroyed by various perturbations, leading to interesting symmetry crossover characteristic to the graphene system [5–8].

There are three different kinds of phonons contributing to electron-phonon interaction. They are long-wavelength acoustic phonons, zone-center optical phonons, and zone-boundary phonons [9]. In particular, optical phonons can be strongly modified by the Fermi-level tuning due to the interaction [10,11]. Inter-layer interaction in bilayer graphene destroys the linear dispersion into an approximate parabolic dispersion [12,13]. In this system, strong asymmetry in the potential of two layers can be introduced by external fields, leading to band-gap opening and strong mixing of symmetric and antisymmetric optical phonons [14,15].

[1] T. Ando, *Physica* E **40** (2007) 213 and references cited therein.
[2] T. Ando and T. Nakanishi, *J. Phys. Soc. Jpn*. **67** (1998) 1704.
[3] T. Ando, T. Nakanishi, and R. Saito, *J. Phys. Soc. Jpn*. **67** (1998) 2857.
[4] T. Ando and H. Suzuura, *J. Phys. Soc. Jpn*. **71** (2002) 2753.
[5] H. Suzuura and T. Ando, *Phys. Rev. Lett.* **89** (2002) 266603.
[6] T. Ando and K. Akimoto, *J. Phys. Soc. Jpn*. **73** (2004) 1895.
[7] T. Ando, *J. Phys. Soc. Jpn*. **75** (2006) 054701.
[8] E. McCann, K. Kechedzhi, V. I. Falko, H. Suzuura, T. Ando, and B. L. Altshuler, *Phys. Rev. Lett.* **97** (2006) 146805.
[9] H. Suzuura and T. Ando, *J. Phys. Soc. Jpn*. **77** (2008) 044703.
[10] T. Ando, *J. Phys. Soc. Jpn*. **75** (2006) 124701.
[11] T. Ando, *J. Phys. Soc. Jpn*. **76** (2007) 024712.
[12] E. McCann and V. I. Falko, *Phys. Rev. Lett*. **96** (2006) 086805.
[13] M. Koshino and T. Ando, *Phys. Rev*. B **73** (2006) 245403.
[14] T. Ando, *J. Phys. Soc. Jpn*. **76** (2007) 104711.
[15] T. Ando and M. Koshino, *J. Phys. Soc. Jpn*. **78** (2009) 034709.